

Ph.D. Dissertation

**Mass spectrum and vacuum structure  
of higher-dimensional non-abelian  
gauge theories with magnetic flux  
background**

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# Abstract

The Standard Model is our best theory that describes the fundamental particles and their interactions. Despite its success, it faces some limitations such as the hierarchy problem, which refers to the unnatural fine tuning required to obtain the mass of the Higgs boson. One promising approach to address this issue involves the idea of extra dimensions. For instance, in Gauge-Higgs Unification models, the Higgs is identified as a scalar originating from an extra-dimensional gauge field. This gives a possible resolution to the hierarchy problem and a new perspective on the origin of electroweak symmetry breaking.

Considering these motivations, we investigate extra-dimensional non-abelian gauge theories as a potential framework for applications in beyond the Standard Model theories. We take non-trivial background configurations of the extra-dimensional gauge fields. A constant background value is related to Wilson line phases, which have flat potential at tree-level and obtain finite effective potentials through quantum corrections. We also introduce a constant magnetic flux in the background configuration. This leads to many desirable features, such as developing chiral fermions having a generation structure in the effective theory. The flux also breaks the translational symmetry of the extra dimensions, giving rise Nambu-Goldstone bosons which remain massless even at one-loop level.

We begin by introducing basic concepts of non-abelian gauge theories on a general  $2N$  extra-dimensional spacetime with non-trivial background configurations. Next, we take a six-dimensional case as an example. The main goal of this discussion is to clarify the complete mass spectra considering a general background configuration and extending the gauge group to  $SU(n)$ . We have shown that tachyonic states appear in this general setup. Thus, using these derivations, we extend to an eight-dimensional spacetime, which is the main work of this thesis. Our objective was to obtain a phenomenologically viable configuration, considering the non-trivial background with flux and Wilson line phases, having no tachyonic states.

We examine the dynamics of the Wilson line phases along directions with no flux background. Hence, they give contributions to masses of low-energy modes, including the potentially tachyonic states. We could find a parameter region of Wilson line phases where no tachyonic modes exist. In this region, by taking an

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$SU(3)$  gauge theory as an example, we calculate the one-loop effective potential for the Wilson line phases. When including matter field contributions, our results reveal the presence of local minima of the potential. In contrast, if we only consider a pure Yang-Mills theory, there are no local minimum points. Taking a minimum as an example, we calculate the mass matrix of fluctuations around it and show that it is positive. Thus, we could obtain a configuration with magnetic flux background that can have viable phenomenological meta-stable vacua.

This thesis is based on the following papers:

- K. Kojima, Y. Okubo and C. S. Takeda,  
“Mass spectrum in a six-dimensional  $SU(n)$  gauge theory on a magnetized torus,”  
[JHEP 08 \(2023\), 083, arXiv:2306.00644 \[hep-th\]](#).
- K. Kojima, Y. Okubo and C. S. Takeda,  
“Vacuum structure of an eight-dimensional  $SU(3)$  gauge theory on a magnetized torus”,  
[Physical Review D, 110 \(2024\), 016028, arXiv:2405.10962 \[hep-ph\]](#).