Ph. D. Dissertation

A new method for calculating the decay rate of the false vacuum at finite temperature by use of the saddle-point approximation

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Abstract

Our universe has undergone several phase transitions to become what it is today. By properly utilizing these transitions, many phenomena observed today can be explained in a relatively simple and elegant manner without introducing new mechanisms. However, these phase transitions occurred in the early universe at finite temperature, making their associated transition rates an intriguing research topic if they are first-order phase transitions.

When calculating the transition rate at finite temperature, the process can be made more efficient by utilizing its relationship with the imaginary part of the free energy put forward in previous research. Furthermore, due to the extensibility of calculations using the path integral approach, the free energy is often reformulated into a path integral expression, which can then be evaluated using the saddle point approximation.

In previous research, the trace in free energy has been treated as equivalent to imposing periodic boundary conditions in the path integral formulation. Thus, calculating the path integral required identifying all classical solutions satisfying periodic boundary conditions and then applying the saddle point approximation to account for contributions around these solutions. However, inconsistencies arise with this method, particularly in the choice of boundary conditions to constrain the paths and their corresponding fluctuations, as well as in the treatment of the zero eigenvalue appearing for particular classical solutions.

In this research, we propose a new calculation method that emphasizes a rigorous evaluation of the integral originating from the trace in the free energy. This approach introduces two key differences compared to conventional methods. First, the trace is explicitly evaluated by performing integration over the endpoints of the paths in the path integral. Second, the saddle point approximation is conducted in a functional space that satisfies Dirichlet boundary conditions, instead of the periodic boundary conditions used in previous studies.

This new approach eliminates the need for the so-called collective coordinate method to handle the zero eigenvalue in the calculation. Moreover, it provides a more intuitive way of calculating the trace in the path integral approach, avoiding the ambiguous application of periodic boundary conditions. Transition rates calculated with our method exhibit expected behavior over a broader range compared to those obtained using conventional approaches.